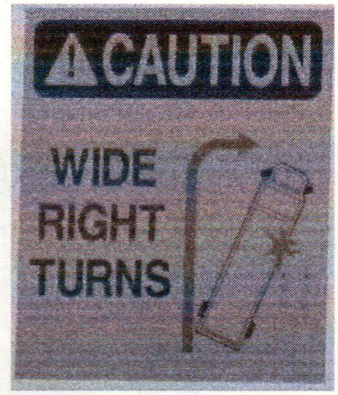
5.6 More Than Right



A Develop Understanding Task

We can use right triangle trigonometry and the Pythagorean theorem to solve for missing sides and angles in a right triangle. What about other triangles? How might we find unknown sides and angles in acute or obtuse triangles if we only know a few pieces of information about them?

In the previous task we found it might be helpful to create right triangles by drawing an altitude in a non-right triangle. We can then apply trigonometry or the Pythagorean theorem to the smaller right triangles, which may help us learn something about the sides and angles in the larger triangle.

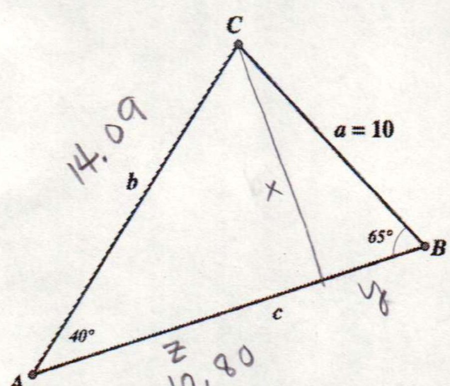
See if you can devise a strategy for finding the missing sides and angles of each of these triangles.

1.

$b \approx 14.09$

$c \approx 15.03$

$\angle C = 75°$



$\sin 65 = \frac{x}{10}$ $\quad 10\sin 65 = x \approx 9.06$

$\cos 65 = \frac{y}{10}$ $\quad 10\cos 65 = y \approx 4.23$

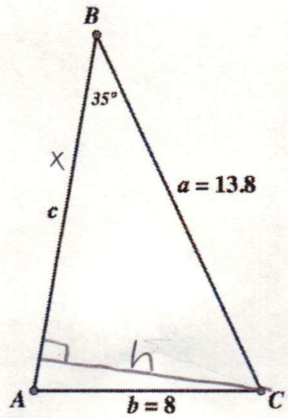
$\sin 40 = \frac{9.06}{b}$ $\quad 9.06/\sin 40 = b \approx 14.09$

$\tan 40 = \frac{9.06}{z}$ $\quad 9.06/\tan 40 = z \approx 10.80$

$c = 10.80 + 4.23 \approx 15.03$

$C = 180 - 65 - 40 = 75°$

2.



$\sin 35° = \frac{h}{13.8}$ $\quad 13.8 \sin 35 \approx 7.92$

$\cos 35° = \frac{x}{13.8}$ $\quad 13.8 \cos 35 \approx 11.30$

$\sin A = \frac{7.92}{8}$ $\quad \sin^{-1}\left(\frac{7.92}{8}\right) \approx 81.9°$

$\cos 81.9 = \frac{y}{8}$ $\quad 8 \cos 81.9 \approx 1.13$

$c = 1.13 + 11.30 \approx 12.43$

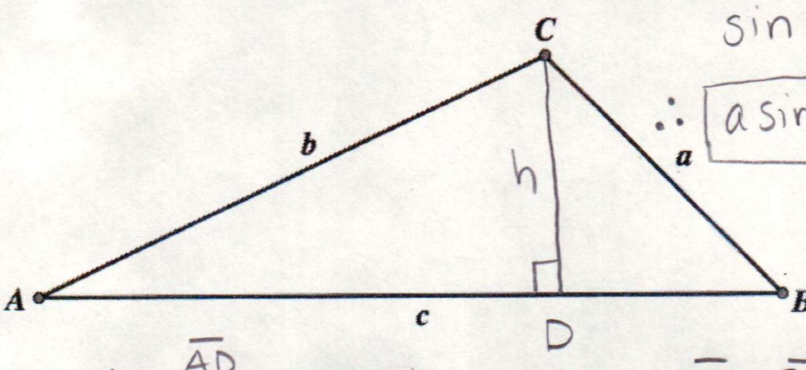
$C = 180 - 35 - 81.9° \approx 63.1°$

3. See if you can generalize the work you have done on problems 1 and 2 by finding relationships between sides and angles in the following diagram. Unlike the previous two problems, this triangle contains an obtuse angle at *C*. Find as many relationships as you can between sides *a*, *b* and *c* and the related angles *A*, *B* and *C*.

$\sin A = \frac{h}{b}$ $\quad h = b \sin A$

$\sin B = \frac{h}{a}$ $\quad h = a \sin B$

$\therefore \boxed{a \sin B = b \sin A = h}$



$\cos A = \frac{\overline{AD}}{b}$ $\quad \overline{AD} = b \cos A$ $\quad \overline{AD} + \overline{BD} = \overline{AB}$

$\cos B = \frac{\overline{BD}}{a}$ $\quad \overline{BD} = a \cos B$ $\quad \therefore \boxed{b \cos A + a \cos B = c}$